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Relationship between the Effects of Productive Failure on University Students and Learner Characteristics

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With the growing importance of statistics education at the university level, productive failure has been identified as an effective instructional strategy for fostering understanding of statistical concepts. However, some studies suggest that its effectiveness may vary depending on learner characteristics. This study focused on attitudes toward mathematics, failure beliefs, and belief in cooperation, and examined the effects of a productive failure-based class on students' problem-solving. The results revealed a significant negative correlation between instrumental motivation to learn mathematics and the quantity of problem-solving, while no significant correlations were found for the other variables. These results suggest that learner characteristics do not have a substantial impact on the effectiveness of productive failure in university-level statistics education, although scaffolding may still be necessary for students with high instrumental motivation to learn mathematics.

Keywords: Productive failure, Conceptual knowledge, Learner characteristics, Statistics education

Introduction

In today's advanced information society, the ability to interpret statistical information is increasingly essential, underscoring the growing importance of statistics education (Otani, 2017). To cultivate this ability, numerous studies have emphasized the significance of understanding the core principles and relationships within statistics (Crooks et al., 2019). For example, in the context of covariance analysis, the core principles and relationships include understanding why it is necessary to control for the effects of covariates in order to compare group means accurately. The knowledge required to understand these principles and relationships is known as conceptual knowledge (Crooks & Alibali, 2014).

In recent years, many studies have demonstrated the effectiveness of an instructional strategy called productive failure in acquiring conceptual knowledge (Loibl et al., 2017). In productive failure, students first attempt to solve complex problems in groups before receiving instruction on the underlying concepts and procedures. Although learners often fail to solve problems initially, this trial-and-error process activates their prior knowledge, which can enhance subsequent learning.

Although productive failure is a promising approach for acquiring conceptual knowledge, its effectiveness depends on meaningful trial-and-error experiences during problem-solving. DeCaro et al. (2015) found that when learners with a strong performance orientation attempt problems that are misaligned with their current skill level, they tend to adopt inefficient strategies, which negatively affects their learning outcomes. This indicates that the effectiveness of productive failure may vary depending on the learner's characteristics. In particular, the PISA surveys have reported that Japanese learners have negative attitudes toward mathematics, failure, and collaborative learning (OECD, 2024, 2019, 2017). Productive failure may presuppose positive attitudes toward mathematics, failure and collaboration. In fact, students in Singapore—where research on productive failure is well established—have been reported to demonstrate high levels of confidence in mathematics (OECD, 2024). This suggests that the effectiveness of productive failure observed in previous studies may have been partly contingent on participants already possessing such positive attitudes toward mathematics, failure, and collaborative learning. Therefore, the impact of productive failure may be influenced to some extent by learners' background characteristics, and similar outcomes may not be readily generalizable to Japanese learners.

However, little research has been conducted on productive failure in the Japanese context. Although Japanese students are reported to perform well in mathematics in international assessments such as PISA, they tend to lack conceptual understanding (Fujimura, 2011). Thus, productive failure may serve as a promising instructional approach to address this shortcoming. Furthermore, Sinha and Kapur (2021a) note that there is a lack of research on the relationship between learner characteristics and the effectiveness of productive failure. Investigating the effects of productive failure in relation to learner characteristics among Japanese students could make two significant contributions: (1) a practical contribution to the specific challenges faced by Japanese learners, and (2) a theoretical contribution by clarifying the relationship between productive failure and individual learner characteristics.

In one of the few studies on productive failure in Japan, Higuchi and Watanabe (2024) implemented a productive failure-based class for high school students and examined learner characteristics that influenced its effectiveness. However, their study did not incorporate group-based problem-solving. Given that Kapur and Bielaczyc (2012) identify group problem-solving as a key component of productive failure, further research on its impact in group settings is essential.

To date, little research has examined the impact of learner characteristics on productive failure-based lessons that involve group-based problem-solving activities (Sinha & Kapur, 2021a). To address this gap, we implemented a productive failure-based class for university students and investigated the influence of learner characteristics on both learning outcomes and group-based problem-solving activities.

Literature Review

Conceptual Knowledge

Many learning and development theories distinguish between two types of mathematical knowledge: "conceptual" and "procedural." In mathematical psychology, conceptual understanding is often referred to as conceptual knowledge, and research has been conducted on its relationship to procedural knowledge (Rittle-Johnson, 2019).

Procedural knowledge refers to knowledge of the steps involved in solving a problem (Rittle-Johnson, 1999). In contrast, Crooks and Alibali (2014) note that conceptual knowledge encompasses "general principles knowledge" and "knowledge of the principles underlying procedures." General-principle knowledge refers to fundamental domains, specific laws, and definitions. Knowledge of the principles underlying the procedures comprises knowledge of why the procedure is valid for the problem and the purpose of each step of the procedure.

In a society where scientific and technological development continues to advance, conceptual knowledge in mathematics is becoming increasingly important, as simple computational procedures can now be performed by computers (National Council of Teachers of Mathematics, 2000). Given the recent advancements in AI, the importance of conceptual knowledge is expected to grow even further. Therefore, it is essential to consider instructional strategies that effectively promote the acquisition of conceptual knowledge.

Productive Failure

In recent years, learning sciences research has significantly focused on the "Problem-Solving followed by Instruction (PS-I)" approach, in which students engage in problem-solving involving previously unlearned concepts before receiving instruction (Sinha & Kapur, 2021a). This contrasts the "Instruction followed by Problem-Solving (I-PS)" approach, where problem-solving follows instruction on an unlearned concept. Many studies report that PS-I is more effective than I-PS for promoting conceptual knowledge acquisition (Loibl et al., 2017; Kapur, 2016).

PS-I typically comprises two phases: an initial problem-solving phase, where students tackle complex problems that entail unfamiliar concepts, and a subsequent instruction phase, where explicit teaching of those concepts occurs. However, the design of the initial problem-solving and subsequent instruction phases varied across studies. In particular, one PS-I approach, known as "Productive Failure" (Kapur & Bielaczyc, 2012), entails providing students with complex problems, prompting them to generate multiple solutions, yet ultimately guiding them to fail in achieving the correct one.

For example, Kapur (2012) compared the effectiveness of productive failure with that of the direct instruction method (Kirschner et al., 2006), which begins with an explanation of the concept and subsequently moves on to problem-solving exercises, in teaching "variance" in mathematics to third-year junior high school students. Students who received instruction via productive failure were requested to develop a method for computing variance without any special support during the problem-solving phase and then received an explanation of the concept of variance during the instruction phase. The results demonstrated that despite failing to develop a method for calculating variance, students who received instruction through productive failure outperformed those who received instruction via direct instruction regarding their scores on post-tests of conceptual knowledge and transfer.

Sinha and Kapur (2021a) performed a meta-analysis of PS-I research to examine the alignment of PS-I class designs with the principles of productive failure and their impact on learning effectiveness in a comparative study of PS-I and I-PS. They identified the following seven criteria as indicators of fidelity to productive failure design principles.

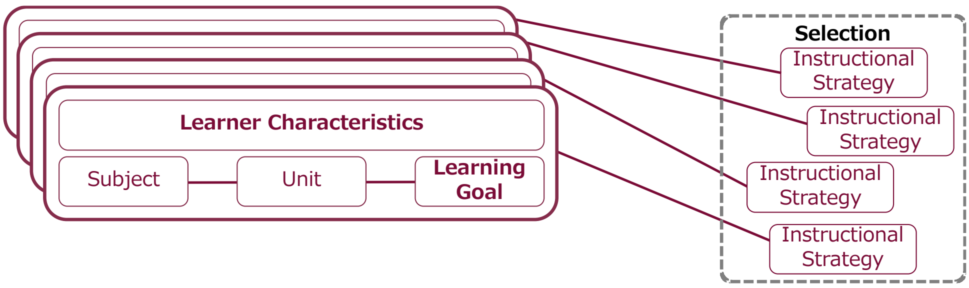
1. Problems that afford multiple representations and solution methods (RSMs)
2. Qualitative or quantitative evidence for multiple RSM generations in the article (e.g., if authors tabulated the quantity/quality of solutions students generated)
3. Affective appeal of the problem considered (e.g., if the problem design comprised story problems within or outside virtual environments, contrasting cases, simulations, and agent-based modeling)
4. Use of group work as the participation structure during the problem-solving phase
5. Instruction that builds on student-generated solutions
6. A socially supportive environment during the problem-solving phase (e.g., whether the problem-solving phase provides a safe space to explore and generate ideas, the social and mathematical norms surrounding the activity, and the provision of affective/motivational support to persist and keep trying)
7. A socially supportive environment during the instruction phase: dialogue-dominant (the teacher asked clarifying questions and/or paraphrased student solutions and/or asked students to elaborate upon each other's ideas and/or engaged students in arguments/conflicts and/or used other kinds of facilitation strategies that enhance student engagement).

Their analysis revealed that the closer an instructional design adhered to productive failure principles, the more effectively PS-I facilitated conceptual knowledge acquisition. Therefore, classes designed based on these principles are likely to acquire conceptual knowledge.

Instructional Design

Even when designing lessons based on productive failure, as with general instruction, it is considered effective to design them based on instructional design theory. Instructional design is a field of models and research that combines methods to foster activities' effectiveness, efficiency, and appeal (Suzuki, 2005). Gagné et al. (2005) explained that Instructional Design involves aligning the desired outcomes, instructional methods, and student assessments. Instructional methods include instructional strategies. Gagné et al. (2005) emphasize that it is essential to select instructional strategies based on learner characteristics and learning goals. Figure 1 illustrates the process of selecting instructional strategies.

Figure 1.
The process of selecting instructional strategies



According to a meta-analysis by Sinha and Kapur (2021a), productive failure has not shown a significant advantage over instruction followed by problem-solving (I-PS) in the acquisition of procedural knowledge. Therefore, productive failure appears to be particularly effective for fostering conceptual understanding and knowledge transfer. However, there has been limited research on the relationship between learner characteristics and the effectiveness of productive failure (Sinha & Kapur, 2021a). Further investigation is needed to clarify how individual differences influence the outcomes of productive failure.

Learner Characteristics

This section outlines the learner characteristics that were the focus of this study.

Failure Beliefs. A characteristic of Japanese high school students is that they tend to fear failure more than students in other countries, not only in academic settings but also in other areas (OECD, 2019). Nishimura et al. (2017) point out that in learning approaches that leverage failure, such as productive failure, it is essential for learners to have a positive attitude toward failure.

Attitudes Toward Mathematics. Japanese high school students are also known to have negative attitudes toward mathematics (OECD, 2024). Lim and Chapman (2013) state that numerous studies have shown a relationship between attitudes toward mathematics and mathematics achievement. Consequently, it is highly likely that such attitudes influence learning through productive failures. Higuchi and Watanabe (2024) conducted a class on conditional probability using a productive failure approach with second-year high school students. Their findings indicate that self-confidence in mathematics has an impact on the learning outcomes of productive failure.

Belief in Cooperation. Nagahama et al. (2009) emphasize that learners must have a positive perception of collaboration to fully benefit from collaborative learning. They identify three factors that influence perceptions of collaboration: the usefulness of collaboration, individual orientation, and inequity.

The usefulness of collaboration measures the extent to which learners perceive the value of learning in a group. Individual orientation reflects the extent to which learners believe that individual learning is more valuable than group learning. The inequity factor measures the degree to which learners experience worry about potential issues arising in group learning. Given the nature of these factors, a more positive perception of collaboration correlates with higher scores on the collaborative utility factor and diminished scores on both individual orientation and reciprocity.

One of the few studies on productive failure conducted in Japan is by Higuchi and Watanabe (2024), who implemented a productive failure lesson on conditional probability for high school students and examined how two factors—attitudes toward mathematics and failure beliefs—influenced learning outcomes. Their findings indicated that students' confidence in mathematics affected their learning outcomes. However, their lesson design did not include group-based problem solving.

Kapur and Bielaczyc (2012) identified group problem solving as a key feature of productive failure. According to Kapur (2024), solving problems in groups facilitates the activation of prior knowledge through sharing ideas with peers, and also enhances the recognition of knowledge gaps—both of which are central mechanisms of productive failure (Loibl et al., 2017). Furthermore, cognitive load theory suggests that when students are engaged in solving complex problems, working in groups can help reduce cognitive load and thereby support learning (Kirschner et al., 2018). Therefore, in contrast to Higuchi and Watanabe's (2024) individual work, collaborative learning through group-based productive failure might promote deeper learning regardless of learner characteristics.

However, for collaboration learning to be effective, learners need to hold positive perceptions of collaboration (Nagahama, 2009). Nevertheless, it remains unclear whether the perceptions of collaboration directly affect the learning outcomes of productive failures.

Purpose

We implemented a statistics lesson based on productive failure for university students majoring in data science. The aim was to investigate how learner characteristics—specifically, failure beliefs, attitudes toward mathematics, and belief in cooperation—influence learning outcomes in statistics and group activities, including engagement and the quantity and quality of solutions generated during the problem-solving phase.

Methods

Participants and Design

This study was conducted in December 2024 as part of a course titled "Mathematics Education Theory 2" at a private university in Tokyo. The participants were 23 third-year undergraduate students enrolled in a course that was part of a teacher-training program and majoring in data science. Prior to the experiment, the participants were informed that participation in the survey was voluntary, that it would not affect their course grades, and that their privacy would be protected.

The procedure comprised three steps: (1) a pre-questionnaire assessing the learner's characteristics was administered (10 minutes), (2) This was followed by a productive failure-based class session (50 minutes), and (3) a post-questionnaire (5 minutes) and post-test (10 minutes) were conducted.

Learning Material

The class topic was "analysis of covariance." The learning objectives were (1) to explain the relationship between the analysis of covariance and regression lines and (2) to understand the key assumptions of the analysis of covariance, specifically that the regression lines of the two groups are parallel. We selected this content because, although many of the participating students had likely encountered the topic in their university statistics courses, we assumed that they were unlikely to have acquired conceptual knowledge of covariance analysis unless they had studied it independently. In fact, only one group was able to correctly solve the problems presented in this class.

The problem used during the productive failure activity was developed based on the design principles of productive failure (Sinha & Kapur, 2021a). This problem is illustrated in Figure 1. According to Sinha and Kapur (2021a), a productive failure problem should be complex and challenging, allow for multiple solutions and representations, and engage learners by drawing on their prior knowledge. Moreover, the problem should be presented in a context that captures students' interest and encourages engagement. In this class, students were given a problem pertaining to the analysis of covariance within a scenario; students took on the role of junior high school mathematics teachers. The problem allowed for diverse solutions and representations, such as comparing bar graphs and scatter plots or comparing post-test scores and score gains from pre- to post-test. We expected that by generating these solutions, students would be able to comprehend the advantages of using the analysis of covariance over other methods. The optimal solution involved plotting the regression lines for the pre- and post-test scores of each group and comparing the intercepts. However, we anticipated that this would be challenging for students unfamiliar with the analysis of covariance.

Figure 2.
The Problems used in this practice (some tables omitted)

You are a junior high school mathematics teacher who wants to improve your regular math lessons. To explore a new approach, you decided to compare a lesson using manga (comic-style explanations) with a traditional lesson.
First, you administered a pre-test (worth 100 points) to 20 students in Class A and 20 students in Class B. Then, you taught Class A using a manga-based lesson and taught Class B using a regular, traditional lesson. After the lessons, you gave both classes a post-test (also worth 100 points) to assess their understanding of the topic. The test scores are shown in the table on the right.
Using Google Spreadsheets, analyze the data to determine which class performed better and provide mathematical reasoning for your conclusion. Do not rely on only one method. Try to come up with as many different approaches as possible to compare the performance of the two classes. For each method, clearly explain what you calculated and why it helps evaluate which class was more effective.

Group A: class using manga			Group B: normal class		
Student	Pre-test	Post-test	Student	Pre-test	Post-test
A1	10	52	B1	62	66
A2	29	50	B2	54	75
A3	34	72	B3	75	81
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
A19	32	73	B19	54	60
A20	22	65	B20	56	68

The practical class was designed based on the principles of productive failure (Sinha & Kapur, 2021a). Students worked in small groups—seven groups of two or three groups of three—to solve the problem collaboratively for over 25 minutes each, using a shared Google Spreadsheet. Throughout the activity, the instructor repeatedly emphasized that students should explore more than one solution by considering multiple possibilities. Students were also informed that failure to reach a correct solution was acceptable because the concepts had not yet been formally introduced and that the goal was to engage in a meaningful trial-and-error process. Although students were not prohibited from using Google search during problem-solving, the use of generative AI was explicitly disallowed.

Following the problem-solving activity, a 25-minute instructional phase was conducted. The instructor explained the concept of analysis of covariance in a brief lecture (5 minutes). In this lecture, we explained that by using the data from the problem presented to the students, we could determine the regression lines for each class, with the post-test scores as the dependent variable and the pre-test scores as the explanatory variable. We then showed that comparing the intercepts allows us to control for the influence of pre-test scores. Since the focus was on conceptual knowledge of covariance analysis, we did not go into the specific calculation procedures. Next, the students remained in their original groups to discuss the assumption of covariance. After some groups shared their ideas with the class (e.g., "It cannot be used unless the sample sizes of the two groups are equal"), the instructor addressed common misconceptions and presented the correct assumption—that analysis of covariance is not appropriate when the two regression lines are not parallel (5 minutes). Subsequently, the students were asked to compare the advantages and disadvantages of the solutions they generated using the correct method (10 minutes). Finally, they compared the characteristics of the analysis of covariance with a method focusing on the difference between pre- and post-test scores (5 minutes).

This instructional phase was designed in accordance with the principles of productive failure (Sinha & Kapur, 2021a), particularly "Instruction that builds on student-generated solutions" and "A socially supportive environment during the instruction phase: dialogue-dominant." Therefore, teacher-led explanations were kept to a minimum, and sufficient time was allocated for students to reflect on their own solutions as well as those presented by the instructor. The total duration of the instructional phase was 25 minutes, which is comparable to or longer than the instructional durations reported in several previous studies on productive failure (e.g., Decaro et al., 2023; Loibl & Leuders, 2024).

Data Collection

Pre-questionnaire. To clarify how the characteristics of Japanese students impact the class using productive failure, we measured students' failure beliefs, attitudes toward mathematics, and beliefs about collaboration using a 5-point Likert scale (1 = "Strongly disagree" to 5 = "Strongly agree").

The failure beliefs in problem-solving were assessed using a scale developed by Nishimura et al. (2017). It comprised two factors: Utilization-possibility of failure (5 items, e.g., "Failure is something that will benefit you in the future") and Fearfulness toward failure (5 items, e.g., "Failure is embarrassing").

Attitudes toward mathematics were measured using items from the PISA survey (National Institute for Educational Policy Research, 2013), with the response format adapted from a 4-point to a 5-point scale to maintain consistency across measures. The scale comprised three factors: Intrinsic motivation to learn mathematics (4 items, e.g., "I look forward to math class"), Instrumental motivation to learn mathematics (4 items, e.g., "Mathematics is worth working hard at because it will be useful for the job I want to do in the future"), and Mathematics self-concept (5 items, e.g., "I get good grades in math").

Beliefs in Cooperation were assessed using a scale developed by Nagahama et al. (2009), which included 18 items in total: Collaborative Utility Factor (9 items, e.g., "I can achieve better results by working together than by working alone"), Individual Orientation Factor (6 items, e.g., "It takes a long time when we all discuss things together"), and Inequity Factor (3 items, e.g., "Cooperation is for people who cannot do their work well").

Worksheet. To evaluate the quantity and quality of student-generated solutions and representations during the problem-solving phase, we analyzed student entries made on Google Spreadsheet. The scoring criteria for quantity were based on Kapur and Bielaczyc's (2012) framework for evaluating solution diversity in productive failure contexts. For instance, if a group created and interpreted a bar graph of post-test scores, this was counted as a unique solution or representation.

In addition, the quality of solutions was evaluated based on the following criteria:
(1) Whether a scatter plot was created for each group
(2) Whether a regression line was plotted for each group, and
(3) Whether students provided an appropriate interpretation of the regression lines

Post-questionnaire. To assess the quality of the learning experience, we measured students' engagement. Items pertaining to behavioral engagement (4 items, e.g., "I studied really hard in this class") were based on Umemoto and Tanaka (2012), while items pertaining to emotional engagement (5 items, e.g., "I enjoyed this class") were adapted from Umemoto et al. (2016). Owing to procedural errors, emotional engagement was assessed using only four items.

Post-test. To evaluate the learning outcomes, we administered a post-test whereby students were asked to determine which of the two diet programs was more effective. Students were presented with four different regression line scenarios, each representing the relationship between initial weight and weight loss for each program. Students were asked to judge which program appeared more effective or if no clear decision could be made for each scenario. Each item was worth one point.

Guidelines for Analysis

We conducted Shapiro–Wilk tests for each item to assess normality. The results indicated that some variables did not follow a normal distribution. Therefore, we opted to use nonparametric tests.

To clarify the impact of learner characteristics on learning outcomes and group activities, we conducted a Spearman's rank correlation analysis between three learner characteristics—attitudes toward mathematics, failure beliefs, and belief in cooperation—and five outcome variables: post-test scores, behavioral engagement, emotional engagement, quantity of solutions and representations, and quality of solutions.

Results

The analysis was conducted after excluding data with missing or incomplete responses (19 valid responses; missing rate: 17.39%). The Jamovi statistical software was used (ver. 2.4.12).

Descriptive statistics and Reliability

Descriptive statistics and McDonald's omega for each variable are presented in Table 1. Sufficient internal consistency was confirmed for all variables. In addition, only one group received full marks regarding the quality of their solutions. This implies that the problem could not be easily solved, even if it was searched online.

Table 1
Descriptive Statistics, including means (M) and standard deviations (SD) on variables

Variables	*M*	*SD*	*ω*
Intrinsic motivation to learn mathematics	3.93	0.58	.62
Instrumental motivation to learn mathematics	4.01	0.88	.88
Mathematics Self-concept	3.33	0.68	.72
Utilization-possibility of failure	4.17	0.53	.77
Fearfulness toward failure	2.06	0.85	.85
Collaborative utility	4.24	0.49	.87
Individual orientation	3.04	0.70	.72
Inequity	1.90	0.69	.73
Behavioral engagement	4.34	0.44	.70
Emotional engagement	4.26	0.59	.85
Solution quantity	3.84	1.17	—
Solution quality	0.95	1.03	—
Post-test	3.79	0.42	—

n = 19

Impact of learner characteristics on learning outcomes and group activities

We conducted Spearman's rank correlation analysis to examine the relationships between three learner characteristics—attitudes toward mathematics, views on failure in problem-solving situations, and perceptions of collaborative work—and five outcome variables: post-test scores, behavioral engagement, emotional engagement, quantity of solutions and representations, and quality of solutions. Table 2 presents the results of correlation analyses.

Table 2
Correlation between learning outcomes and learner characteristics

	Intrinsic motivation	Instrumental motivation	Self-concept	Utilization-possibility of failure	Fearfulness toward failure	Collaborative utility	Individual orientation	Inequity
Behavioral engagement	.04	.33*	-.12	.32	-.23	.36	.01	-.37
Emotional engagement	-.19	.10*	-.01	.29	-.26	.19	-.22	-.26
Solution quantity	-.28	-.51*	-.27	-.26	.40	-.31	-.19	.17
Solution quality	-.08	-.27*	-.37	-.26	.27	.19	-.36	-.15
Post-test	-.35	-.26*	-.02	-.09	.04	-.10	.13	.00

$n = 19$ $^{*}p < .05$

The analysis revealed a strong negative correlation between instrumental motivation in mathematics and the quantity of solutions generated. No significant correlations were observed among the remaining variables.

Discussion

The analysis revealed a strong negative correlation between instrumental motivation and the quantity of solutions. No other significant correlations were observed. This finding reinforces the findings of Decaro et al. (2015), who investigated the effects of achievement motivation on learning outcomes through PS-I tasks with students in grades two to four. They found that students with higher levels of mastery orientation employed more advanced problem-solving strategies and acquired greater conceptual knowledge, whereas those with stronger performance orientation tended to adopt inefficient strategies, which hindered their acquisition of procedural knowledge. In other words, learners driven by extrinsic motivation, such as performance orientation, are more likely to rely on inefficient problem-solving strategies. Therefore, it is plausible that students with high instrumental motivation in mathematics, typically associated with extrinsic motivation, also persist with less effective strategies, limiting the variety of solutions they generate.

In contrast, behavioral engagement exhibited a moderate positive correlation, although this was not statistically significant. This may be because the recognition of one's gaps in knowledge during problem-solving fostered student motivation and fostered greater effort during the instructional phase.

Therefore, for students with high instrumental motivation, it may be necessary to provide scaffolding that encourages the use of a wider range of problem-solving strategies. Previous research has explored the integration of scaffolding into productive failure. For example, Holmes et al. (2014) reported that generic scaffolding was effective in helping university students develop a deeper understanding of the components of formulas in lessons on least-squares methods and weighted averages. By incorporating such scaffolding, even students with high instrumental motivation may improve their approach during the problem-solving phase, thereby promoting more effective learning. However, few studies have examined how the effects of scaffolding in productive failure vary depending on learner characteristics, indicating a need for further research (Sinha & Kapur, 2021b).

No significant correlations were observed between other learner characteristics and either engagement or post-test scores. Given the generally high post-test scores, this indicates that productive failure effectively supported the acquisition of conceptual knowledge in statistics among university student, regardless of their individual learner characteristics. This finding contradicts the results of Higuchi and Watanabe (2024), who reported that mathematical confidence influences the effectiveness of lessons based on productive failure. One possible explanation for this discrepancy is that, unlike Higuchi and Watanabe (2024), the present study implemented the problem-solving phase of productive failure in groups. Group-based implementation may have facilitated engagement with challenging tasks, thereby reducing the impact of individual learner characteristics.

Perceptions of collaborative work did not significantly affect learning outcomes or group activities. This may be attributed to the inherent difficulty of problems addressed in the context of productive failure. Generally, perceptions of collaboration can influence the effectiveness of group learning, particularly when learners feel that their individual work is more efficient, which may discourage cooperation. However, in productive failure tasks, learners are compelled to collaborate to tackle challenging problems. Consequently, perceptions of collaboration may have had limited effect in this context. Nonetheless, a moderate negative correlation was observed between individual orientation and post-test scores, although this was not statistically significant. This could be due to the limited sample size, indicating the need for further investigations with a larger cohort to confirm the reproducibility of the findings.

Descriptive statistics revealed a ceiling effect in the post-test scores, indicating that many students had already achieved the learning objectives of the lesson. However, this ceiling effect makes it difficult to evaluate the effectiveness of productive failure. In this study, no significant correlations were found between post-test scores and learner characteristics. Therefore, the results of this study suggest that productive failure may be effective even for students who lack confidence in mathematics, in contrast to students in Singapore who typically exhibit high mathematical confidence. However, this does not necessarily imply that learner characteristics have no impact on the effectiveness of productive failure. Future research should improve the post-test to increase its discriminatory power and conduct further investigations.

The instructional design in this study was grounded solely in the principles of productive failure, rather than tailored to a particular mathematical topic. As such, the findings may have applicability to other mathematical units and even across subject areas. Nonetheless, as noted above, this study has several limitations, including the ceiling effect in the post-test and the small sample size.

Conclusion

We aim to clarify the impact of learner characteristics on learning outcomes in classes that use productive failure. Two key findings emerged:

- Students exhibiting high instrumental motivation in mathematics tended to produce lower-quantity solutions to their problems.
- Overall, productive failure was effective in promoting conceptual understanding of the concept of covariance analysis for university students majoring in data science regardless of individual differences in learner characteristics.

However, this study had several limitations, including a small sample size, lack of control over participants' prior knowledge, and a ceiling effect observed in the post-test. Therefore, it is premature to generalize these findings to all university students. Further practice and analysis with larger sample sizes are necessary.

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